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BRIEF COMMUNICATION

COMPUTATION OF THE EFFECT OF HEAT ADDITION ON INTERFACIAL SHEAR IN BUBBLY FLOW

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INTRODUCTION

The more sophisticated computer programs for the analysis of the transient behavior of two-phase flow systems use models which require estimates of the rate of mass, energy and momentum transfer between phases. McFadden *et al.* (1981) proposed the so-called "dynamic slip" model which uses the momentum equations to obtain the dynamic behavior of the phase velocity difference. The model is derived by writing the momentum balances for vapor and liquid phases, subtracting the resultant equations and solving for the time derivative of the relative velocity between the phases. For steady-state conditions, where the relative velocity time derivative is zero, Crawford *et al.* (1985) were able to substantially simplify the dynamic slip equation. For vertical flow, only the liquid is in contact with the wall, and the term containing the friction between vapor and wall is set to zero. Further, the terms involving the spatial derivatives of the gas and liquid velocities were determined to be negligible. After expressing the interfacial drag in terms of the parameter $C_D A_{FG}$, where C_D is the interfacial (liquid–gas) drag coefficient and A_{FG} is the interfacial (gas–liquid) surface area per unit volume (length^{-1}), Crawford *et al.* (1985) showed that for vertical flow under steady-state conditions, $C_D A_{FG}$ can be related to the flow parameters by

$$C_D A_{FG}\left[\frac{\bar{\rho} V_{FG}^2}{(8\rho_G)\epsilon(1-\epsilon)}\right] = -\left(\frac{1}{\rho_L}-\frac{1}{\rho_G}\right)\frac{dP}{dz} - \frac{f_{WL} V_L^2}{2D(1-\epsilon)}, \qquad [1]$$

where

D = tube diameter,
f_{WL} = friction factor between the liquid and the wall,
P = pressure (force/area),
z = axial distance (length),
V_G, V_L = mean velocities of the gas and liquid, respectively,
$V_{FG} = V_G - V_L$,
ϵ = void (vapor) fraction

and

$\rho_G, \rho_L, \bar{\rho}$ = gas, liquid and average densities, respectively (mass/length^3).

V_L, V_{FG} and $\bar{\rho}$ may be computed when the void fraction, pressure drop and quality are known. With the foregoing quantities and values of f_{WL} from standard correlations, [1] may be used to determine $C_D A_{FG}$. Equation [1] is particularly useful for evaluation of interfacial shear when experimental values of ϵ and quality are available since otherwise a relationship between ϵ and quality is required.

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It should be noted that computationally the dynamic slip model has little value for most steady-state flow calculations. The expectation is that the values of $C_D A_{FG}$, computed as a function of ϵ and G (total mass flux), from steady-state measurements will be used in the original dynamic slip model equations for the computation of the relative velocity between phases during various transient situations. For rapid transients, steady-state relationships between ϵ and quality are usually in error. It is implicitly assumed that, at constant ϵ and G, there is essentially no difference between the steady-state and transient values of $C_D A_{FG}$.

To aid modelers of two-phase flow transients, Ying & Weisman (1988) used experimental measurements of void fraction and quality with refrigerant 113 to determine values of $C_D A_{FG}$ for adiabatic flow in 2.5 cm tube as a function of void fraction and mass velocity. Values were determined for both upflow and downflow. They also used literature data to compute $C_D A_{FG}$ values for the steam–water system in adiabatic flow. Ying & Weisman (1988) presented a computational model for interfacial shear in adiabatic flow which compared well with the $C_D A_{FG}$ values derived from the experimental data.

In a subsequent paper, Ying & Weisman (1989)† presented experimentally derived values for $C_D A_{FG}$ for diabatic flow of refrigerant 113 in a 2.5 cm tube. They found that, in the bubbly flow region, the values of $C_D A_{FG}$ with heating were appreciably above those for adiabatic flow. This was true both for upflow and downflow. The increase in the peak values of the drag ranged from 20 to 70%. It was also found that the peak value of $C_D A_{FG}$ occurred at a higher void fraction than in adiabatic flow.

In the present paper, we extend the model of Ying & Weisman (1988) for interfacial shear in adiabatic flow to diabatic conditions. The extension is limited to the bubbly flow region.

MODEL FOR INTERFACIAL SHEAR IN ADIABATIC FLOW

Modeling the interfacial drag in bubbly flow begins with the determination of the bubble shape and the size distribution. This information is then used to determine the bubble rise velocities and drag coefficients. The absolute velocities of the vapor and liquid phases, V_G and V_L, are then determined enabling the value of $C_D A_{FG}$ to be computed from

$$C_D A_{FG} = \frac{\sum N_0 C_{Di} A_i f_i (V_{Gi} - V_L)|V_{Gi} - V_L|}{(V_G - V_L)|V_G - V_L|}, \qquad [2]$$

where

N_0 = total number of bubbles per unit volume,
C_{Di} = drag coefficient for bubbles with equivalent diameter d_i,
A_i = interfacial area of bubble with diameter d_i,
f_i = number fraction of bubbles with diameter $d_i \pm \alpha$,
$V_L = G(1-x)/[\rho_L(1-\epsilon)]$,
$V_G = Gx/(\rho_G \epsilon)$
V_{Gi} = velocity of bubbles in size category i

and

x = quality.

Note that [2] simply equates the total drag on all of the bubbles to the total drag represented in a manner consistent with [1].

Based on the work of Ben-Yosef *et al.* (1975) on bubble size distribution, it is assumed that the bubbles follow the log-normal distribution which is expressed as

$$f(r) = \left(\frac{1}{2\pi\beta}\right)^{1/2} r^{-1} \exp\left\{-\left(\frac{\beta^2}{2}\right)\left[\ln\left(\frac{r}{\Omega}\right)^2\right]\right\}, \qquad [3]$$

†The caption for figure 6 of Ying & Weisman (1989) should be labeled as describing "downflow" rather than "upflow".

where r is the equivalent bubble radius, Ω is a scale factor and β is a shape parameter which accounts for the skewness of the distribution. Ying & Weisman (1989) assumed β was a function of void fraction. Further, to account for the fact that large numbers of very small bubbles are created at high mass fluxes, β was also taken to be a function of the mass flux at high masss fluxes. The value of β is given by

$$\beta = \begin{cases} (0.625 - 0.52\epsilon) & \text{for } G \leqslant G_s \\ (0.625 - 0.5\epsilon)\left(\dfrac{G}{G_s}\right) & \text{for } G \geqslant G_s \end{cases}, \quad [4]$$

where $G_s = 4.9 \times 10^6\ \text{kg/m}^2\,\text{h}$ ($1 \times 10^6\ \text{lb/h ft}^2$). Since the mean of the log-normal distribution is given by

$$r_m = \Omega \exp\left(\frac{\beta^2}{2}\right), \quad [5]$$

the value of Ω is fixed when the mean radius, r_m, is determined.

At low void fractions, the mean bubble diameter ($d_m = 2r_m$) is computed from Hinze's (1955) equation

$$d_m = 0.725\left(\frac{\sigma}{\rho_L}\right)^{3/5}\left(\frac{p}{m}\right)^{-2/5}, \quad [6]$$

where σ is the surface tension and p/m = mechanical power dissipated/mass of fluid. It was found that Hinze's equation gives smaller average bubble sizes as the void fraction increases, which is contrary to the experimental observations which indicate that the size of the bubble increases as void fraction increases. The mean bubble diameter, d_m, was therefore calculated by obtaining the Hinze diameter at $\epsilon = 0.05$ and multiplying this value by the agglomerating factor. The values of the agglomerating factors, which were determined as those leading to the best fit of the experimental interfacial shear data, are given in figure 4 of Ying & Weisman (1988). Separate curves are used for upflow and downflow.

Bubble size alone is not sufficient to determine bubble rise velocity. Consideration must also be given to the bubble shape. Bubbles were considered to belong to one of three broad shape categories: spherical, ellipsoidal or spherical cap. A regime map adapted from that proposed by Clift *et al.* (1978) was used for this purpose [see figure 5 of Ying & Weisman (1988)]. Bubble categories are therefore determined in accordance with the values of the bubble Reynolds number (Re) and Eotvos number (Eo).

The interfacial area, A_i, of a spherical bubble is given as

$$A_i = 3\left(\frac{v_b}{r}\right), \quad [7]$$

where v_b is the volume of a bubble and r is its radius. However, for other bubble shapes the approach of Mishima & Ishi (1984) was followed. The interfacial area is then calculated as

$$A_i = 3\left(\frac{v_b}{r_{Sm}}\right), \quad [8]$$

where r_{Sm} is the Sauter mean radius. In order to calculate r_{Sm}, the shape factor for each geometry must be considered. For ellipsoidal bubbles, the shape factors are correlated in terms of the aspect ratio. In the range of interest, the time-averaged aspect ratio (the ratio of the maximum vertical dimension, h, to the maximum horizontal dimension, $2r$) can be predicted from an equation proposed by Wellek *et al.* (1966):

$$\frac{h}{2r} = \frac{1}{1 + 0.163\text{Eo}^{0.757}}$$

and

$$\text{Eo} = \frac{g\,\Delta\rho d^2}{\sigma} = \text{Eotvos number}. \quad [9]$$

The shape factor r_v/r_{Sm} for the ellipsoidal shape can be calculated from an equation given by Mishima & Ishi (1984) after noting that $r_v = (3v_b/4\pi)^{1/3}$. For cap bubbles, the aspect ratio $(h/2r_v)$ can be expressed in terms of the wake angle θ' and predicted in accordance with the suggestion of Mishima & Ishi (1984).

Unhindered rise velocities for single bubbles in each of the three categories are obtained in accordance with Peebles & Garber (1953) for spherical bubbles, Grace *et al.* (1976) for ellipsoidal bubbles and Hetsroni (1982) for cap bubbles. These single bubble rise velocities are then corrected for the presence of the tube wall and other bubbles. The wall effect was accounted for using Harmathy's (1960) correction, while the effect of other bubbles (void fraction effect) was taken as that suggested by Zuber (1964).

The drag coefficients in the spherical and ellipsoidal regimes are calculated from the equations of Braues (1973). However, very small bubbles are considered to exert no drag. Husain & Weisman (1978) found that the mean bubble size in dispersed (homogeneous) flow was $\leqslant 45\%$ of the Hinze (1955) diameter evaluated from a total mass flow rate equal to 7.9×10^6 kg/m^2 h (1.6×10^6 lb$_m$/ft^2 h). Based on the above, bubbles with diameter equal to our less than this value were considered to be traveling along with the liquid with no drag.

In the spherical cap regime, the drag coefficient is taken as 8/3 for upward vertical flow (Hetsroni 1982). Since no previous study of drag coefficients for a cap bubble in downward flow was found, Ying & Weisman (1988) estimated the drag coefficient of a cap bubble in downward flow by using the fact that the drag coefficient for a half-cylinder is equal to 1.2 if the flow strikes the cap region of the half-cylinder directly. However, the drag coefficient for the flow striking the flat side of the half-cylinder is equal to 1.7. The drag coefficient for a cap bubble in downward flow is taken to be $8/3 \times (1.2/1.7)$, where $(1.2/1.7)$ is the correction factor for the flow direction.

To obtain absolute bubble velocities, use was made of the so-called "drift flux" approach. There, the absolute velocity of the bubble in the flowing stream is expressed as the algebraic sum of the rise velocity in a stagnant liquid and a term proportional to the mean velocity of the mixture, i.e.

$$V_G = C_0 V_m + u_b, \qquad [10]$$

where

$$V_m = V_{GS} + V_{LS}.$$

The quantities V_{GS} and V_{LS} are the superficial gas and liquid velocities (assuming each phase flowing alone), respectively. Note that, in downward flow, V_m, V_{GS} and V_{LS} are negative.

The situation is complicated by the experimental observation that, in any given assemblage of bubbles, the observed rise velocity of the larger bubbles in upward flow can only be accounted for by using a larger value of C_0 than that used for the smaller bubbles. However, in downward flow, a smaller value is required to account for the differences between the behaviour of the large and small bubbles in an assemblage. The bubbles were therefore divided into three categories according to their sizes. The bubbles in categories j have an absolute velocity, V_{Gj}, given as

$$V_{Gj} = C_{0j} V_m + u_b. \qquad [11]$$

Values for C_0 in upward flow have been determined by Zuber *et al.* (1967) and others as being in the range 1.1–1.6 Crawford *et al.* (1986) indicated that in downward flow, $C_0 \leqslant 1.0$. By comparison of observed and calculated relative velocities, the values shown in table 2 of Ying & Weisman (1988) were obtained. The average absolute velocity of the bubbles is calculated as

$$V_G = \sum f_i V_{Gi}. \qquad [12]$$

The mean relative velocity $(V_G - V_L)$ is then obtained after computing V_L from

$$V_L = \frac{G(1-x)}{\rho_L(1-\epsilon)}. \qquad [13]$$

The computation of V_G via [12] is open to question. It might be expected that an evaluation of V_G in which the contribution of a given category is weighted in accordance with the volume fraction in the category would provide a better estimate. However, Ying (1985) found that relative velocities computed via [12] gave the best agreement with her observed slip ratios.

Once the mean relative velocity of the vapor, $(V_G - V_L)$, is calculated, nearly all the quantities required to obtain the product $C_D A_{FG}$ via [2] are available. However, neither N_0 nor x have been determined. The number of bubbles per unit volume, N_0, may be determined by first finding the void fraction corresponding to an arbitrary N_0. The correct N_0 is determined by multiplying the assumed value by the ratio of the actual to calculated void fraction.

As assumed value of the quality is required to obtain an initial value of V_L. The vapor velocity resulting from the model calculations is used to obtain a revised value for x. The calculation continues iteratively until successive values of x are in satisfactory agreement.

Ying & Weisman (1988) compared values of $C_D A_{FG}$ computed from the foregoing model to their own data for refrigerant 113 (in upflow and downflow) and literature data for steam–water systems at two different pressures. Good agreement between the experimentally derived and computed values was obtained.

EXTENSION OF THE ADIABATIC MODEL TO DIABATIC CONDITIONS

When boiling occurs at a heated wall, vapor bubbles are constantly being introduced at the heated surface. Under flow conditions, these bubbles are generally considerably smaller than the equilibrium size reached under adiabatic conditions. It is these small bubbles which cause the increase in interfacial shear and the movement of the peak interfacial shear to a higher void fraction.

The mean radius of the bubbles introduced at the heated wall was based on the widely accepted approach of Levy (1967) who equated buoyancy, friction and surface tension forces at the point of bubble departure. At high velocities, Levy (1967) concluded that the buoyancy term was negligible. However, at the lower velocities examined by Ying & Weisman (1989), buoyancy forces may not be excluded. We therefore use the Levy (1967) equation as modified by Ying & Weisman (1986) to include the buoyancy force. They obtained for r_b, the mean bubble radius:

$$r_b = \frac{C_1 \left(\dfrac{G}{\dfrac{\tau_W}{D_h}} \right)^{0.5}}{1 + C_2 \left[\dfrac{g(\rho_L - \rho_G)\tau_W}{g_c D_H} \right]^{0.5}}, \qquad [14]$$

where

C_1 = constant = 0.015,
C_2 = constant = 0.1,
D_H = hydraulic diameter,
τ_W = wall shear stress,
g = gravitational acceleration
g_c = gravitational conversion factor.

When the small bubbles leave the heated wall, the bubbles collide and interact. They eventually reach the equilibrium size which would be seen in adiabatic flow at the same mass flux and void fraction. It was assumed that the average bubble size would approach the equilibrium size exponentially and could be expressed as

$$r_m = r_L\, e^{-kz/D} + r_E(1 - e^{-kz/D}), \qquad [15]$$

where

r_m = mean bubble radius at position z,
r_L = radius computed from modified Levy equation, [14]
r_E = equilibrium bubble size for adiabatic conditions as computed from [6] and the agglomerating factors of figure 4 of Ying & Weisman (1988),
k = empirical constant

and

z = distance between point at which bubble is generated and point of interest.

To compute the void fraction–shear relationship under the uniform heat addition rates of Ying (1985), the tube was divided into a series of axial segments. In the inlet segment, the tube contains only bubbles generated in that segment. These have a mean bubble size given by [14] using a z corresponding to half the length of the axial segment. However, the bubble sizes are distributed around the mean in accordance with Ben-Yosef's log-normal distribution [3]. We therefore divide the bubbles into a number of size categories (generally ten) and determine the fraction of bubbles in each category. For each size category, the individiual bubble volumes, velocities, interfacial areas and drag coefficients are computed following the procedures of the adiabatic model.

For a given heat flux and heated perimeter, Q_v, the volumetric flux of vapor per unit time generated in the given segment, can be computed as

$$Q_v = q'' \frac{p_h \Delta z}{h_{LG}(\rho_G)}, \qquad [16]$$

where

q'' = heat flux (energy/area),
p_h = heated perimeter (length),
Δz = length of axial segment

and

h_{LG} = heat of vaporization.

Since

$$Q_v = n_j \sum_i f_i (v_b)_i, \qquad [17]$$

where

n_j = total number of bubbles generated in segment j per unit time,
f_i = fraction of bubbles in size category i

and

$(v_b)_i$ = average volume of a bubble in size category i,

the total number of bubbles generated in the segment per unit time can be obtained. However, to obtain the void fraction and interfacial shear, the number of bubbles present in a unit volume is needed. For any given size category, N_i, the number of bubbles per unit volume at the exit of the segment is

$$N_i = \frac{n_j f_i}{(u_b)_i a}, \qquad [18]$$

where

$(u_b)_i$ = velocity of a bubble in category i

and

a = flow area.

Hence

$$\epsilon = \sum_i N_i (v_b)_i \qquad [19]$$

and

$$C_D A_{FG} = \sum_i \frac{C_{Di} N_i A_i f_i (V_{Gi} - V_L)|V_{Gi} - V_L|}{(V_G - V_L)|V_G - V_L|}. \qquad [20]$$

As noted in discussing [2], the above formulation simply equates the total interfacial drag calculated to the interfacial drag defined in terms of the average vapor and liquid velocities.

Subsequent axial segments contain the bubbles generated in that segment as well as bubbles produced in all the previous segments. We proceed as we did for the first segment, but first consider the bubbles generated in each of the axial segments independently. The value of z is determined for each group, the mean radius determined, and the parameters for each size category computed as for the first segment. To avoid iteration, the agglomerating factor was based on the void fraction obtained from the experimental data for the quality and mass flow being considered.

To compute the void fractions and interfacial drag, the combined effect of the bubbles generated in the segment at the given location and all those at a lower location must be considered. We therefore have at the exit of segment k:

$$a_j = \sum_j \sum_i N_{ij}(v_b)_{ij} \quad [21]$$

and

$$C_D A_{FG} = \frac{\sum_j \sum_i C_{Dij} N_{ij} A_{ij} f_{ij}(V_{Gij} - V_i)|V_{Gij} - V_L|}{(V_G - V_L)|V_G - V_L|}, \quad [22]$$

with V_G and V_L being determined from x and ϵ (see the definition following [4]). The index "ij" indicates bubbles in size category "i" generated in segment "j". In computing the values for $C_D A_{FG}$, we used the previously indicated approach of assuming zero drag for bubbles having sizes as small or smaller than those seen in homogeneous flow.

Ying & Weisman (1988) showed that, in downward flow, the onset of the unstable region corresponded to the conditions where the absolute velocity of the gas was zero. For computational purposes, this condition was taken as that corresponding to an absolute bubble velocity of <0.5 cm/s. Bubbles with lower absolute velocities were assumed to be circulating in small eddies with a zero velocity relative to the liquid. Their drag coefficient was therefore taken as zero.

RESULTS OF COMPUTATIONS

A series of computations were carried out for the range of conditions examined by Ying & Weisman (1989). The refrigerant 113 tests they reported upon were conducted at 2 bar with mass velocities ranging from 1.5×10^6 to 4.9×10^6 kg/m^2 h and a heat flux of 3–6 $\times 10^4$ W/cm^2. The value of k, the empirical constant in [14], was varied from about 0.008 to 0.8. Values of $k \leqslant 0.01$ cause the interfacial shear to fall below that for adiabatic flow, as only a large number of small bubbles

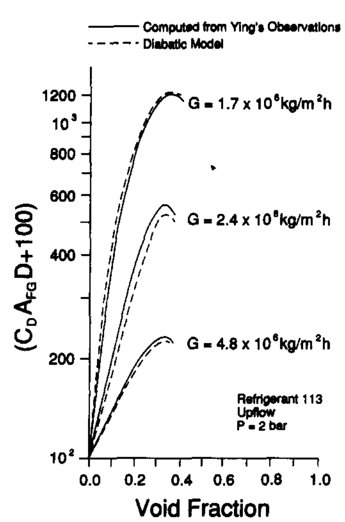

Fig. 1. Comparison of observed and calculated values of $C_D A_{FG}$ for diabatic upflow.

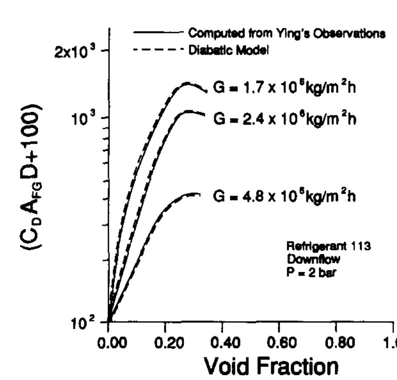

Fig. 2. Comparison of observed and calculated values of $C_D A_{FG}$ for diabatic downflow.

remain present and many are in the size range where zero shear is assigned. Values of $k > 0.01$ increase the value of the shear, but values of $k \geqslant 0.4$ produce very little additional effect.

The best results were obtained with $k = 0.035$. This value was found to be satisfactory for all mass fluxes in both upflow and downflow. Nearly as good results were obtained with $k = 0.025$ or $k = 0.04$. Figures 1 and 2 compare the computed results using $k = 0.035$ with the $C_D A_{FG}$ values derived by Ying & Weisman (1988) from experimental measurements. To make the plot non-dimensional, $C_D A_{FG}$ has been multiplied by D. It may be seen that good agreement is obtained. Both the magnitude of $C_D A_{FG}$ and the location of the peak value are satisfactorily predicted.

CONCLUSION

The proposed revision of Ying & Weisman's (1988) model for interfacial shear in bubbly flow appears to provide a satisfactory explanation of the experimental observations of increased interfacial shear under diabatic conditions. The observed increase in shear and shift in the peak shear are consistent with the change expected by the addition of small bubbles at the heated wall. However, the revised model is not entirely general since the value of k, the empirical constant in the equation used to describe the variation of bubble size with distance from the point of bubble generation, can be regarded as valid only for refrigerant 113 over the range of conditions examined by Ying & Weisman (1989). Additional data covering other fluids and pressures are needed before any generalizations can be made.

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